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Review

A guide to multi-objective optimization for ecological problems with an application to cackling goose management

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ABSTRACT

Choices in ecological research and management are the result of balancing multiple, often competing, objectives. *Multi-objective optimization* (MOO) is a formal decision-theoretic framework for solving multiple objective problems. MOO is used extensively in other fields including engineering, economics, and operations research. However, its application for solving ecological problems has been sparse, perhaps due to a lack of widespread understanding. Thus, our objective was to provide an accessible primer on MOO, including a review of methods common in other fields, a review of their application in ecology, and a demonstration to an applied resource management problem.

A large class of methods for solving MOO problems can be separated into two strategies: modelling preferences pre-optimization (the *a priori* strategy), or modelling preferences post-optimization (the *a posteriori* strategy). The *a priori* strategy requires describing preferences among objectives without knowledge of how preferences affect the resulting decision. In the *a posteriori* strategy, the decision maker simultaneously considers a set of solutions (the Pareto optimal set) and makes a choice based on the trade-offs observed in the set. We describe several methods for modelling preferences pre-optimization, including: the bounded objective function method, the lexicographic method, and the weighted-sum method. We discuss modelling preferences post-optimization through examination of the Pareto optimal set. We applied each MOO strategy to the natural resource management problem of selecting a population target for cackling goose (*Branta hutchinsii minima*) abundance. Cackling geese provide food security to Native Alaskan subsistence hunters in the goose's nesting area, but depredate crops on private agricultural fields in wintering areas. We developed objective functions to represent the competing objectives related to the cackling goose population target and identified an optimal solution first using the *a priori* strategy, and then by examining trade-offs in the Pareto set using the *a posteriori* strategy. We used four approaches for selecting a final solution within the *a posteriori* strategy; the most common optimal solution, the most robust optimal solution, and two solutions based on maximizing a restricted portion of the Pareto set. We discuss MOO with respect to natural resource management, but MOO is sufficiently general to cover any ecological problem that contains multiple competing objectives that can be quantified using objective functions.

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Contents

1. Introduction

Ecological decisions that require balancing multiple objectives are pervasive. Examples include endangered species management (e.g., maximizing species persistence while minimizing cost; Maguire et al., 1987), managing harvested species (e.g., maximizing cumulative harvest while maintaining population objectives; Johnson et al., 1997), and choosing statistical models to infer ecological processes (i.e., maximizing model fit while minimizing model complexity; Williams, 2016). When a decision maker has multiple competing objectives, a solution that simultaneously optimizes each objective does not exist; improving one objective results in a trade-off from another. Solving multi-objective decision problems requires incorporating decision-maker preferences among objectives into the decision problem (either explicitly or implicitly) to reach a compromise solution.

The process of solving multiple objective problems generally consists of identifying or soliciting the objectives of the decision maker, identifying potential actions, assessing the potential actions (or the predicted outcome of the actions) with respect to each objective, and making a choice. Scientific investigation can be used to predict the result of potential actions, but science alone is insufficient to address competing objectives because incorporating preferences among objectives requires value-based judgment (Holland-Bartels and Pierce, 2011). Some actions might result in obtaining one objective, while other actions might obtain another objective. Ultimately, a decision maker can implement one management action (or suite of actions). Thus, how can we formally combine a quantification of objectives, with objective preferences to select a final, optimal decision? *Multi-objective optimization* (MOO) is a sub-field of multi-criteria decision making that provides a formal mathematical framework for explicitly incorporating objectives and objective preferences to evaluate decisions. In contrast to other multi-criteria decision making methods (e.g., multi-criteria decision analysis), MOO is well suited for handling many objectives, and many (potentially infinite) alternative actions.

We outline the MOO framework and describe two strategies for solving MOO problems. Each strategy incorporates objective preferences into the decision problem. The strategies differ in the order in which preferences are incorporated; the first strategy (the *a priori* strategy) incorporates preferences pre-optimization and the second (the *a posteriori* strategy) incorporates them post-optimization. To demonstrate an application of MOO, we apply both strategies to a common natural resource management problem: selecting a *population target* for an animal population that affects multiple stakeholders differently. We define the population target as the abundance of animals that resource managers aspire to obtain and maintain through time. Our resource management problem was motivated by the management of cackling geese (*Branta hutchinsii minima*) across their range. Cackling geese nest on the coastal plain of the Yukon Kuskokwim (YK) Delta, Alaska. They constitute an important food source for Native Alaskan subsistence hunters. Ecosystem stability, the satisfaction of recreational hunters, and other non-consumptive users also depend on them. In their wintering area in Oregon and Washington (primarily in the Willamette Valley), cackling geese congregate on private agricultural fields and eat crops, resulting in loss of agricultural yield for landowners. The population target for the past 20+ years was 250,000 birds, however, the population target is currently being debated. Thus, we examine selecting a population target in a MOO framework. Multi-objective optimization is general, spanning many disciplines, and strategies used to solve MOO problems provide a framework for making defensible, transparent choices for natural resource management, and ecological decisions in general.

2. The multi-objective optimization problem

Multi-objective optimization assumes a decision maker can quantify the value of a decision with respect to the decision maker's objectives. Examples of objectives that have been explicitly quantified in natural resource management include: minimizing the probability of extinction (Maguire et al., 1987; Ewen et al., 2015; Larkin et al., 2016), maximizing the expected cumulative harvest of a hunted species (Johnson et al., 1997), maximizing the probability of successful population establishment of re-introduced species (Converse et al., 2013), maximizing biodiversity (Arponen et al., 2005; van Teeffelen and Moilanen, 2008; van Teeffelen et al., 2008; Cabeza et al., 2010; Tsai et al., 2015), maximizing habitat suitability (Williams, 1998; Holzkämper et al., 2006; Groot et al., 2007; Zsuffa et al., 2014), and maximizing habitat protection (Kennedy et al., 2008). A function that quantifies the value of the potential actions θ from a set of possible choices of actions Θ relative to an objective is termed an objective function (Keeney and Raiffa, 1976; Williams et al., 2002). Objective functions are inherently subjective because they are used to quantify the aim or interest of a decision maker (Hennig and Kutlukaya, 2007; Williams and Hooten, 2016). For consistency with MOO literature, we denote the objective function using $f(\theta)$ (notation definitions are also reported in Table 1 for reference). Objective functions are synonymous with loss functions, utility functions, or reward functions described in other fields (Williams et al., 2002; Berger, 2013; Williams, 2016; Williams and Hooten, 2016). The set of actions a decision maker can consider (Θ) can be either discrete or continuous. A specific action in the set of Θ is represented by θ. The value of the objective function (or utility) for a specific action is represented by $f(\theta)$. When a decision maker has one objective to maximize, and the objective function is unimodal, the decision maker can simply choose the value for θ that optimizes the objective function $f(\theta)$ (Fig. 1A). Decisions become

Table 1
Notation and definitions of components in multi-objective optimization.

Notation	Description
θ	An action or choice a decision maker can choose. Bold $\boldsymbol{\theta}$ implies decisions for >1 variable.
θ^*	An optimal solution.
Θ	The set or list of actions from which the decision maker can choose. Bold $\boldsymbol{\Theta}$ is the potential combinations of potential actions when a decision maker must make choices for >1 variable.
$f_i(\theta)$	Individual objective function that describes the value of each choice $\theta \in \Theta$.
$f(\boldsymbol{\theta})$	A set of multiple objective functions that depend on (potentially many) choices $\boldsymbol{\theta}$.
$g(\boldsymbol{\theta})$	Inequality constraints.
$h(\boldsymbol{\theta})$	Equality constraints.
Feasible design space	The choices of $\boldsymbol{\theta}$ that meet the constraints.
Feasible criterion space	The values of $\boldsymbol{f}(\boldsymbol{\theta})$ for the feasible design space.

difficult when decision makers must consider more than one objective. A single optimal solution for multiple competing objective functions does not exist without compromise. There are many (possibly infinite) solutions that represent trade-offs among competing objectives. Multi-objective optimization is concerned with methods for choosing among these trade-off solutions.

The MOO problem is defined as:

$$\begin{aligned} & f(\boldsymbol{\theta}^*) = \text{optimum}_{\theta} f(\boldsymbol{\theta}), \\ & \text{where } f(\boldsymbol{\theta}) = (f_1(\boldsymbol{\theta}), f_2(\boldsymbol{\theta}), \ldots, f_k(\boldsymbol{\theta})), \\ & \text{such that } g_j(\boldsymbol{\theta}) \le c_j, \quad j = 1, b, \ldots, J, \\ & \text{and } h_l(\boldsymbol{\theta}) = d_l, \qquad l = 1, 2, \ldots, L, \end{aligned} \tag{1}$$

where $f_i(\boldsymbol{\theta})$ represent the k different, potentially competing objective functions, $f(\boldsymbol{\theta})$ is a set of the different objective functions, and g_j and h_l represent J inequality constraints and L equality constraints, respectively (Hwang and Masud, 1979). The elements of the vector of variables $\boldsymbol{\theta}$ are known formally as *design variables* (Marler and Arora, 2004). $\boldsymbol{\theta}$ are the combination of choices the decision maker can choose for inputs into the decision problem. The set of possible design variables from which a decision maker can choose ($\boldsymbol{\Theta}$) is termed the *feasible design space* and is defined by all potential combinations of choices of $\boldsymbol{\theta}$ that meet the constraints (i.e., $\{\boldsymbol{\theta} | g_j(\boldsymbol{\theta}) \le c_j, j = 1, 2, \ldots, J, h_l(\boldsymbol{\theta}) = d_l, l = 1, 2, \ldots, L\}$). The constraints limit the potential combinations of choices by formally considering items such as budgetary constraints, legal mandates, etc. The *feasible criterion space* includes the values of $f(\boldsymbol{\theta})$ for each $\boldsymbol{\theta}$ in the feasible design space (i.e., $\{f(\boldsymbol{\theta}) | \boldsymbol{\theta} \in \boldsymbol{\Theta}\}$).

Consider an example to clarify notation and concepts. Assume a decision maker wants to maximize $f_1(\theta)$ and $f_2(\theta)$. The functions of $f_1(\theta)$ and $f_2(\theta)$ are general but might represent various natural resource management problems (e.g., satisfaction of two

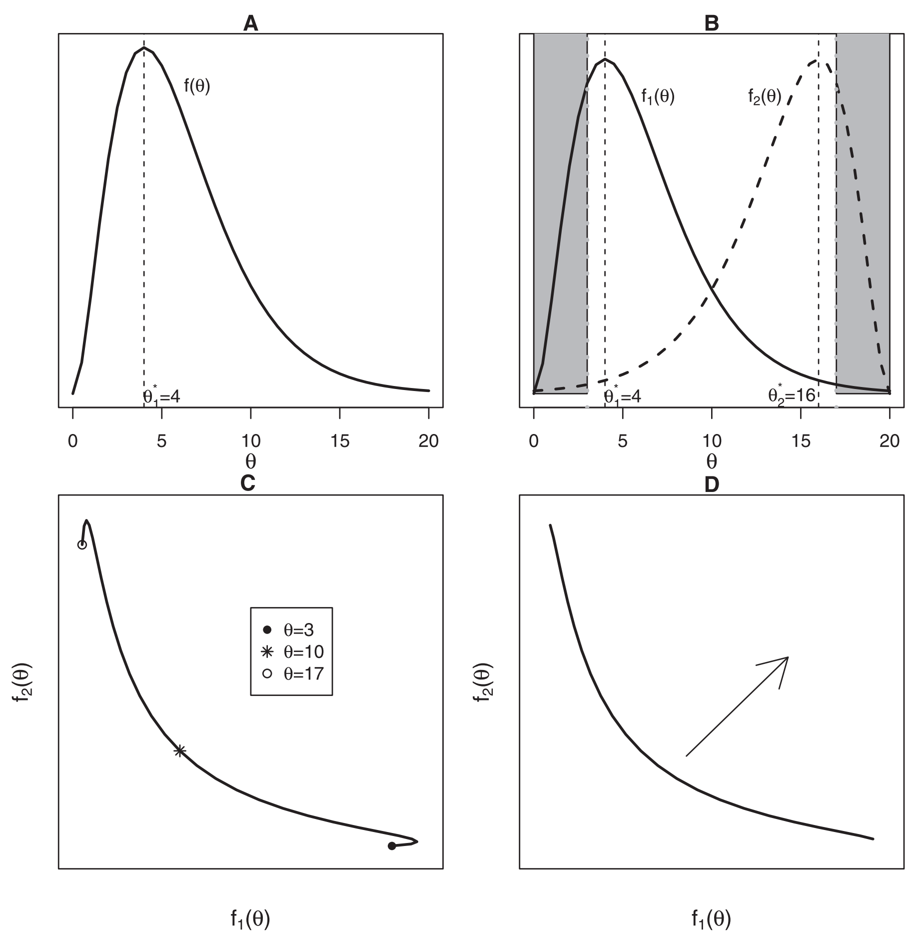

Fig. 1. A: Single objective function $f_1(\theta)$ with optimal solution $\max_\theta(f_1(\theta)) = 4$. B: Multiple objective functions and feasible design space (i.e., possible choices for θ in the domain [3–17]; white area). The optimal values for each objective are: $\max_\theta(f_1(\theta)) = 4$ and $\max_\theta(f_2(\theta)) = 16$. Since $\theta_1^* \ne \theta_2^*$ no value of θ simultaneously optimizes $\boldsymbol{f}(\theta)$. C: Feasible criterion space for multi-objective optimization problem (i.e., $\{\boldsymbol{f}(\theta) | 3 \le \theta \le 17\}$). D: Plot of the Pareto set (i.e., efficiency frontier or Pareto frontier), in which the dominated solutions in C were removed. Values towards the top right of the graph are preferred. Each solution on the Pareto frontier is Pareto optimal and represents a trade-off between objective 1 and objective 2.

opposing stakeholders with respect to possible densities of wolves in a management unit). Attainment of each objective depends on only one design variable $\theta \in \Theta$ (e.g., the possible densities of wolves that could be considered; Fig. 1B). Assume Θ includes the potential choices [0–20]. Also assume the inequality constraints: $g_1(\theta) = \theta \le 17$ and $g_2(\theta) = \theta \ge 3$. These constraints limit the choices of θ such that $3 \le \theta \le 17$ and the feasible design space includes values θ in the domain [3–17]. That is, although it might be desirable to one stakeholder to have densities of wolves <3 wolves per unit, managers might not want densities this low because the population might become extirpated (Fig. 1B). Smaller values of θ imply higher attainment of $f_1(\theta)$ and lower attainment of $f_2(\theta)$. The feasible criterion space consists of the 2×1 vectors $[f_1(\theta), f_2(\theta)]'$ for all $\theta \in$ [3–17]. The feasible criterion space can be viewed graphically (when $k \le 3$ and θ includes one variable) by plotting the paired values of $f_1(\theta)$ and $f_2(\theta)$ against each other on opposing axes (Fig. 1C). Optimal attainment of $f_1(\theta)$ and $f_2(\theta)$ (i.e., $\theta = \theta_1^*$ and $\theta = \theta_2^*$) occur at 4 and 16, respectively (Fig. 1B). No choice of θ simultaneously optimizes $f_1(\theta)$ and $f_2(\theta)$ (i.e., $\theta_1^* \ne \theta_2^*$); a single solution to the MOO problem does not exist. The decision maker must include additional information to reduce the set of potential solutions to a single solution; the additional information required is a decision maker's preferences among objectives.

3. Pareto optimal solutions and specification of preferences

Pareto optimality is a concept in MOO in which optimality is defined with respect to trade-offs that are required to improve an objective (Deb, 2001). A Pareto optimal solution is any solution where there are no other candidate solutions that improve achievement of at least one objective without hindering the achievement of another objective. More rigorously, a Pareto optimal solution is an action $\boldsymbol{\theta}^* \in \boldsymbol{\Theta}$ in which there is no other action $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ such that both $f(\boldsymbol{\theta}) \le f(\boldsymbol{\theta}^*)$, and $f_i(\boldsymbol{\theta}) < f_i(\boldsymbol{\theta}^*)$ for at least one function i (note: we use the minimum, without loss of generality; Deb, 2001; Marler and Arora, 2004). A MOO problem with competing objectives has a large (potentially infinite) set of Pareto optimal solutions. The Pareto set (or Pareto frontier or efficiency frontier) is the set of Pareto optimal solutions. The Pareto set excludes all choices that are dominated by at least one other solution (Deb, 2001). A dominated solution is a solution in which there exists another solution that is as good or better for all objectives, and better for at least one objective. Consider the example in Fig. 1. Any value of θ between 4 and 16 is a Pareto optimal solution. For all $4 < \theta < 16$, to improve $f_2(\theta)$ requires a trade-off from $f_1(\theta)$. Likewise, to improve $f_1(\theta)$ requires a trade-off from $f_2(\theta)$. Note that decisions for $\theta < 4$ and $\theta > 16$ are dominated solutions; both objectives can be improved simultaneously by increasing or decreasing θ, respectively.

Despite its applicability for numerous ecological applications, Pareto optimality has been used in few applications of ecological decision making. Examples include, Reynolds and Ford (1999), who used Pareto optimality as a method for evaluating ecological process models. Similarly, Williams (2016) framed model selection methods (e.g., AIC) in terms of Pareto optimality. Bijleveld et al. (2012) used Pareto optimality to design a benthic monitoring program. Rothley et al. (1997) described variation of insect behavior as differing Pareto optimal solutions for the foraging objectives of survival and growth, and predator avoidance. Similarly, Schmitz et al. (1998) presented Pareto optimality as an extension of optimal foraging theory and examined Pareto optimal solutions for objectives of nutrient maximization and time minimization of foraging snails. Natural resource management examples include Kennedy et al. (2008), who identified Pareto optimal solutions for forest fuels treatments that reduce fire impact on a forest. Kindler (1998), Tsai et al. (2015), Zhou et al. (2015), and Zhou et al. (2015) described Pareto optimality for water resource management. Polasky et al. (2005) identified Pareto optimal land-use patterns with respect to persistence of various species and on market-oriented economic returns. Groot et al. (2007, 2010), Mouysset et al. (2011), and Groot and Rossing (2011) identified Pareto optimal solutions for spatial planning of multi-functional agricultural landscapes. Schröder et al. (2008), using the concepts of Pareto optimality, plotted cost of management actions vs. ecological consequences to quantify the trade-offs between conservation needs and economic constraints for managing semi-natural grassland communities.

Methods for solving (1) can be separated into two strategies: specification of preferences pre-optimization (the *a priori* method) or post-optimization (the *a posteriori* method; Deb, 2001; Marler and Arora, 2004).

3.1. *The* a priori *strategy*

When decision makers specify preferences pre-optimization they are attempting to identify the choice in the Pareto set that most closely aligns with their *a priori* perceptions of the importance of each objective. The information used to assign preferences can include qualitative or quantitative values obtained from a variety of methods ranging from personal opinion to formal theoretical development (e.g., Saaty, 1988; Mustajoki et al., 2005; Williams, 2016). After preferences are assigned pre-optimization, the Pareto optimal solution associated with those preferences is identified. The identification of the Pareto solution associated with a set of preferences occurs using various mathematical functions (Table 2). These functions typically take one of two forms; using a composition of objective functions or constraining the feasible criterion space. A composition of objective functions combines the multiple objective functions into a single objective function. This typically requires specifying weights that represent the relative importance of each of the multiple objective functions. The optimum of the resulting composite function reflects the Pareto optimal solution associated with the selected weights/preferences. Ecology examples using a composition of functions include, for example: Joubert et al. (1997) who combined objectives related to water supply and biodiversity, Converse et al. (2013) who combined several objectives for the reintroduction of whooping cranes (*Grus americana*), and Forzieri et al. (2009) who combined objectives for detecting trees using high-resolution laser scanning. Constraining the feasible criterion space results in an optimization of (1) that occurs over a single objective function (e.g., Johnson et al., 1997). As an example of the *a priori* method, we consider three different functions for assigning preferences pre-optimization to identify an optimal solution, following Marler and Arora (2004): the bounded objective function method, the lexicographic method, and the weighted-sum method. The bounded objective function method and the lexicographic method constrain the feasible criterion space. The weighted-sum method is a composition of functions. These three methods, or combinations of them, are sufficiently general to cover a diverse array of ecological decision problems. We have also included several other methods in Table 2 for reference.

The bounded objective function method assigns preferences for k objective functions by constraining $k-1$ objective functions to preferred ranges of values, then optimizes the final objective function within the constrained space (Marler and Arora, 2004). This effectively reduces the number of objective functions to one with a constrained feasible criterion space that meets the preferences (or are in some tolerable range) of all other objectives. The MOO problem in (1) for the bounded objective function method is defined by $f(\boldsymbol{\theta}^*) = \min_\theta f_1(\boldsymbol{\theta})$, such that: $c_{l,i} \le f_i(\boldsymbol{\theta}) \le c_{u,i}$, $i = 2, \ldots, k$, where $c_{l,i}$ and $c_{u,i}$ represent the lower and upper bounds, respectively, for each objective function $i = 2, \ldots, k$. As an example, suppose θ represents the amount of land purchasable for conservation of

Table 2
Standard functions or methods for incorporating decision maker preferences (in the form of objective weights w_i or goals b_j) into multi-objective optimization problems. For more details see included references or Marler and Arora (2004).

Name of function or method	Function (f=)	See for details
Weighted global criterion 1	$\sum_{i=1}^{k} w_i(f_i(\boldsymbol{\theta}))^p, \quad f_i(\boldsymbol{\theta}) > 0 \quad \forall i$	Yu and Leitmann (1974), Zeleny (1982), Chankong and Haimes (1983)
Weighted global criterion 2	$\sum_{i=1}^{k} (w_i f_i(\boldsymbol{\theta}))^p, \quad f_i(\boldsymbol{\theta}) > 0 \quad \forall i$	Yu and Leitmann (1974), Zeleny (1982), Chankong and Haimes (1983)
Weighted global criterion 3	$(\sum_{i=1}^{k} w_i(f_i(\boldsymbol{\theta}) - f_i^o)^p)^{1/p}, \quad f_i(\boldsymbol{\theta}) > 0 \quad \forall i$	Yu and Leitmann (1974), Zeleny (1982), Chankong and Haimes (1983)
Weighted global criterion 4	$(\sum_{i=1}^{k} (w_i^p(f_i(\boldsymbol{\theta}) - f_i^o)^p)^{1/p}, \quad f_i(\boldsymbol{\theta}) > 0 \quad \forall i$	Yu and Leitmann (1974), Zeleny (1982), Chankong and Haimes (1983)
Hierarchical	$f_j(\boldsymbol{\theta}) \le (1 + \frac{\delta_j}{100})f_j(\boldsymbol{\theta}_j^*), \; j = 1, 2, \ldots i-1, \; i > 1, \; i = 1, 2, \ldots k$	Osyczka (1984)
Weighted Tchebycheff	$w_i(f_i(\boldsymbol{\theta}) - f_i^o)$	Marler and Arora (2004)
Augmented weighted Tchebycheff	$w_i(f_i(\boldsymbol{\theta}) - f_i^o) + \rho\sum_{j=1}^{k}(f_j(\boldsymbol{\theta}) - f_j^o)$	Steuer and Choo (1983)
Modified weighted Tchebycheff	$w_i(f_i(\boldsymbol{\theta}) - f_i^o + \rho\sum_{j=1}^{k}(f_j(\boldsymbol{\theta}) - f_j^o))$	Kaliszewski (1987)
Exponential weighted-criterion	$\sum_{i=1}^{k}(e^{pw_i} - 1)e^{pf_i(\boldsymbol{\theta})}$	Athan and Papalambros (1996)
Weighted-product	$\prod_{i=i}^{k}(f_i\boldsymbol{\theta})^{w_i}$	Bridgman (1922)
Goal programming	$\sum_{j=1}^{k}\lvert d_j\rvert, \; d_j = \lVert b_j - f_j(\boldsymbol{\theta})\rVert$	Charnes and Cooper (1977)

an endangered species and ranges between 0 and 2,000 ha. Suppose we have two objective functions (k = 2). Let $f_1(\theta)$ be a function that returns a value of conservation to the endangered species from the amount of purchased land. For example, the function $f_1(\theta) = (1.1 \times 10^6)/(1 + e^{-0.005(\theta-700)})$ is a non-decreasing function that has diminishing marginal returns for the amount of land purchased and is on a similar scale to $f_2(\theta)$ (Fig. 2). Let $f_2(\theta)$ describe the cost of land. Assume the cost of land is linearly related to the size of land and 1 ha of land costs $1000 (Fig. 2). The decision maker cannot simultaneously maximize $f_1(\theta)$ and minimize $f_2(\theta)$ because both are increasing functions of θ. Suppose the decision maker prefers cost ($f_2(\theta)$) be bounded to ≤ 1 million dollars, with 0 as a minimum bound. The optimal solution can be found using the

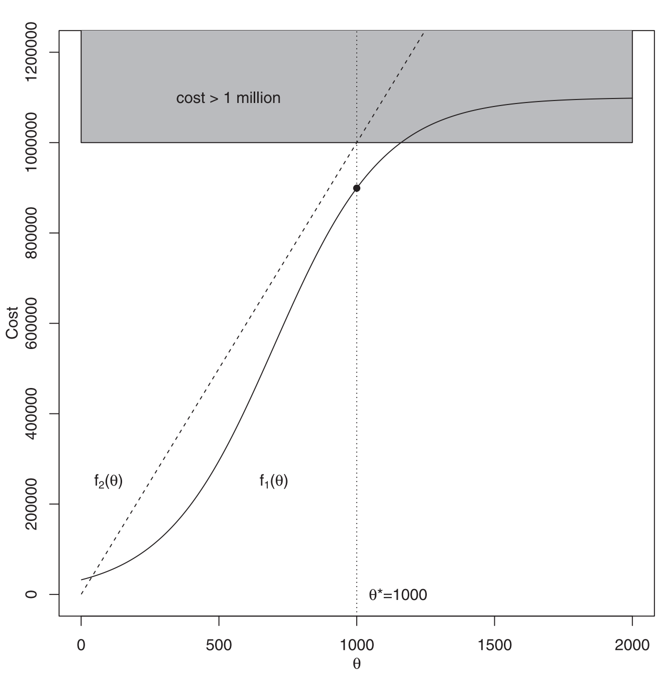

Fig. 2. Example of *Bounded objective function method* with two objective functions representing the value of conservation to an endangered species (solid line) and price (dotted line) for different values of the amount of land that could be purchased (θ). Price was bounded to be between 0 and 1,000,000 dollars. The optimal solution is the value that maximized the value of conservation within the bounds, and occurred at 1000 ha.

bounded objective function method by minimizing $f_1(\theta)$ such that $0 \le f_2(\theta) \le$ 1,000,000. The optimal solution is the maximum of $f_1(\theta)$ in the constrained feasible criterion space determined by $0 \le f_2(\theta) \le$ 1,000,000, and equals 1,000 ha (Fig. 2). Constraints are a natural choice for monetary objectives (or other objectives with explicit boundaries in the feasible criterion space) because they represent realistic budgetary conditions. A limitation of the bounded objective function is that the decision maker must be able to constrain $k-1$ objective functions, which may be difficult in practice.

The lexicographic method involves ordering objectives by importance and subsequent iterative optimization. The intuition behind the lexicographic method is simple. First identify the order of preference of each objective function; the most important function first (Fig. 3). Next identify the optimal solutions for the most important objective function (Fig. 3B). Given multiple optimal solutions to the most important objective function (i.e., multiple global optima), choose the solutions that optimizes the second objective function (Fig. 3C). Continue through the remaining objective functions until one solution is identified (Fig. 3D). The MOO problem for the lexicographic method is defined by $\min_\theta f_i(\boldsymbol{\theta})$, subject to $f_j(\boldsymbol{\theta}) \le f_j(\boldsymbol{\theta}_j^*)$, j = 1, 2, . . . $i-1$, if $i > 1$; i = 1, 2, . . . k, where $f_j(\boldsymbol{\theta}_j^*)$ is the optimal value of the jth objective function. The objective functions are ranked in order of importance from i = 1, 2, . . ., k with i = 1 being the most important. Note that $f_j(\boldsymbol{\theta}_j^*)$ is unique but $\boldsymbol{\theta}_j^*$ is not necessarily unique. Therefore, the decision maker chooses the value of $\boldsymbol{\theta}_j^*$ that optimizes $f_i(\boldsymbol{\theta})$. The optimal solution is sensitive to the ordering of objective functions (Fig. 3). The lexicographical method is useful when a primary objective must be met. For example, when legislative mandates (e.g., in the US the Endangered Species Act and the Migratory Bird Treaty Act) require a decision maker meet one objective; given the first objective is met, optimize with respect to subsequent objectives. The lexicographic method has limited use with an increasing or decreasing primary objective function because the optimum of the primary objective function will occur at a boundary. The hierarchical method (Table 2) is a generalization of the lexicographic method that relaxes the constraints of the lexicographic method to be within some tolerance level (δ) of the optimal solutions of preceding objective functions. That is, $f_j(\boldsymbol{\theta}) \le (1 + (\delta_j/100))f_j(\boldsymbol{\theta}_j^*)$.

The weighted-sum method and its variants are the most common methods for solving MOO problems across disciplines (see: Williams, 1998; Arponen et al., 2005; Holzkämper et al., 2006; van Teeffelen and Moilanen, 2008; van Teeffelen et al., 2008; Roura-Pascual et al., 2009; Cabeza et al., 2010; Converse et al., 2013; Zsuffa

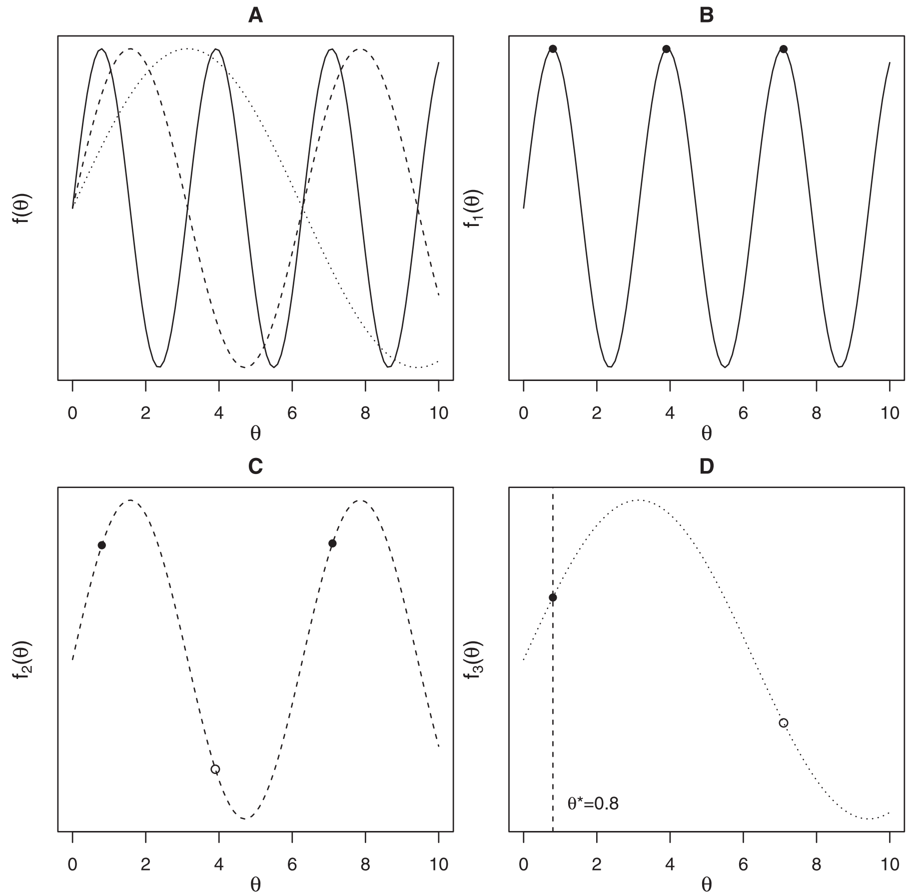

Fig. 3. *Lexicographical method* for solving multi-objective optimization problems. A Three objective functions with no common solution. B Highest priority objective function and three optimal solutions. C Second highest priority objective function with the three optimal solutions of the highest priority solution shown. The open dot represents a sub-optimal solution with respect to the second objective function. D Least important objective function with the two optimal solutions of the second highest priority objective function. The final solution is the closed dot.

et al., 2014; Ewen et al., 2015; Larkin et al., 2016, for ecological applications). A weighted sum of multiple functions is described by:

$$f(\boldsymbol{\theta}) = \sum_{i=1}^{k} w_i f_i(\boldsymbol{\theta}). \tag{2}$$

The weights w_i are chosen by the decision maker to reflect the importance of each objective. Methods for selecting the weights have been the focus of considerable research and discussion in fields ranging from statistics to social science (e.g., Akaike, 1973; Saaty, 1988; Rao and Roy, 1989; Goodwin and Wright, 2004). Weights are often constrained such that $w_i \geq 0$ and $\sum_{i=1}^{k} w_i = 1$ to aid in interpreting weights as relative importance of objectives. The MOO problem using a weighted sum is:

$$f(\boldsymbol{\theta}^*) = \min_{\boldsymbol{\theta}} \sum_{i=1}^{k} w_i f_i(\boldsymbol{\theta}), \tag{3}$$

subject to the constraints $g_j(\boldsymbol{\theta}) \leq c_j$, $j = 1, 2, \ldots, J$, and $h_l(\boldsymbol{\theta}) = d_l$, $l = 1, 2, \ldots, L$. Consider the objective functions used in the lexicographic example (Fig. 3). Instead of the lexicographic method, suppose we apply the weighted-sum method to incorporate objective preferences into the MOO problem. Suppose each objective is equally important such that weights $w_i = 1/3$ for $i = 1, 2, 3$. The resulting objective function is: $f(\theta) = 1/3f_1(\theta) + 1/3f_2(\theta) + 1/3f_3(\theta)$ (Fig. 4A). The optimal solution is $\theta^* = 1$ (*c.f.*, the lexicographic method in which $\theta^* = 0.8$). Had the weights been, for example,

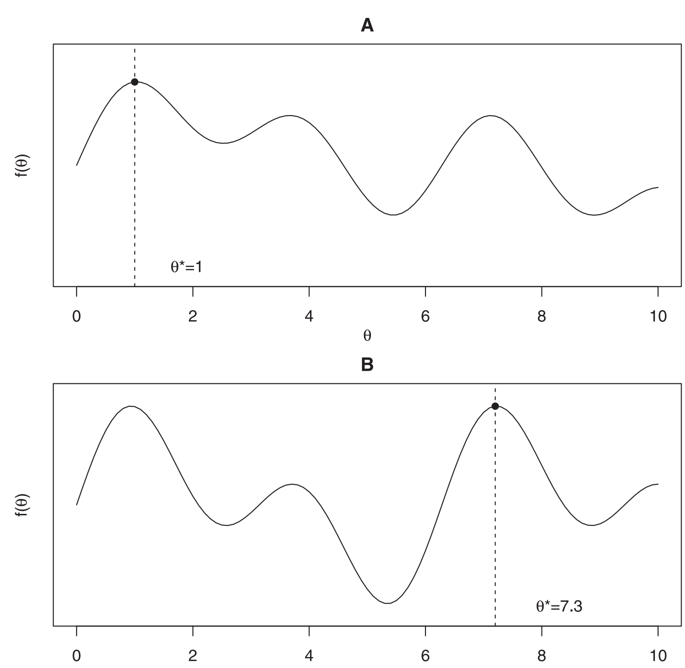

Fig. 4. *The weighted-sum method* applied to the objective functions shown in Fig. 3A. The resulting function to optimize was (A) $\theta^* = \max(1/3f_1(\theta) + 1/3f_2(\theta) + 1/3f_3(\theta))$ and (B) $\theta^* = \max(1/2f_1(\theta) + 1/2f_2(\theta) + 0f_3(\theta))$.

$w_1 = 1/2, w_2 = 1/2, w_3 = 0$, the optimal solution would have been $\theta^* = 0.73$ (Fig. 4B). The weighted-sum method has several advantages that contribute to its widespread use. The intuition is simple, it provides flexibility sufficient to cover a diverse array of problems, and it provides a method for examining the second strategy to MOO, selecting preferences post-optimization (through multiple optimizations over many different combinations of weights). Often, the real challenge with the weighted-sum method is agreeing on a specific set of values of the weights.

The preceding three examples of specifying preferences pre-optimization are intuitively simple, yet cover a diverse array of applications. For problems in which none of the above methods are sufficient to reduce the problem to a solution, combinations of the methods could be applied. Consider a problem with four objectives: minimize cost, maximize species persistence, maximize non-hunting recreation, and maximize hunting recreation. Cost could be constrained at the onset of a problem by identifying the available budget. If species persistence must be obtained the lexicographic method could be used to constrain the decision space. Finally, non-hunting and hunting recreation could be reduced to a single objective using the weighted-sum method. The decision maker could optimize the combined recreation objective within the constrained decision space determined by cost and meeting the species persistence objective. The resulting choice would keep cost under budget, meet the species persistence objective, and maximize the combined non-hunting/hunting recreation objective, given the constraints. Alternative functions for specification of preferences pre-optimization are reported in Table 2. Each of these methods must meet certain criteria to ensure the resulting solution is part of the Pareto optimal set. Marler and Arora (2004) provide a thorough discussion of the requirements for Pareto optimality for the three methods described here and additional methods in Table 2.

Solutions to MOO problems using the *a priori* method are sensitive to the decision maker's choice of preferences and constraints. The level of sensitivity is unknown when preferences are set. After a solution is identified, decision makers' should consider sensitivity analyses to examine the robustness of the final solution to small changes in weights or preferences.

3.2. *The* a posteriori *strategy*

Selecting Pareto solutions using the *a posteriori* strategy is similar to an exhaustive sensitivity analysis described in the previous paragraph. To select a Pareto optimal solution using the *a posteriori* method, the decision maker first identifies as many Pareto optimal solutions within the Pareto set as possible (or every Pareto optimal solution if exact methods are available). Several methods exist for identifying Pareto optimal solutions including: an iterated weighted-sum method, evolutionary algorithms (Deb, 2001), physical programming (Messac et al., 2001; Messac and Mattson, 2002), normal boundary intersection methods (Das and Dennis, 1997, 1999), and normal constraint methods (Messac et al., 2003). The iterated weighted-sum method involves solving the optimization in (3) repeatedly, for different values of weights $\boldsymbol{w}$ (usually in equal increments from 0 to 1 for each w_i). Choosing the size of increment is itself a trade-off in objectives. Smaller increments requires more computational cost, but can proved a better depiction of the Pareto optimal set. The iterated weighted-sum method is the most common optimization technique, and is applicable for many ecological applications (but see Athan and Papalambros, 1996; Das and Dennis, 1997, for limitations). Evolutionary algorithms require users to first generate a *population* of initial solutions. The population of initial solutions is evaluated based on some criteria (e.g., dominance). The best solutions are selected over the others and placed in the *mating pool*. New solutions (offspring) are spawned in the mating pool through the stochastic exchange of traits among the parent solutions in the mating pool. After new solutions are spawned, they are evaluated and the process continues until termination criteria are met resulting in a population of Pareto optimal solutions. Thus, evolutionary algorithms have the computational advantage that a population of Pareto optimal solutions can be produced in one run of an algorithm (as opposed to a single solution). There is an entire field of research known as evolutionary multiobjective optimization which refers to solving MOO problems using evolutionary algorithms (Deb, 2001; Coello et al., 2007).

When a decision maker identifies the Pareto set they obtain information on the trade-offs required for each potential choice $\boldsymbol{\theta}$. The decision maker uses the knowledge of the resulting trade-offs among Pareto optimal solutions (and other information relevant to the problem) to choose a solution from the Pareto set. The choice implies a decision maker's preferences among objectives because each Pareto optimal solution is associated with a set of preferences. Thus, the main difference between pre- and post-specification of preferences is the former requires describing the relative preferences among objectives without any knowledge of the consequences. The latter uses consequences to aid the decision (e.g., such as a decision that is robust across a large range of objective weight values).

When a Pareto set is identified in a MOO problem the decision maker is left with the task of choosing among the Pareto set to obtain a final decision. The advantage of Pareto optimization is that the decision maker first reduces the potential solutions to Pareto optimal solutions. Then the decision maker identifies the trade-offs inherent in the Pareto set. After the trade-offs are identified, any number of methods can be used to make a choice. For example, applying constraints (as is done for the bounded objective function method), identifying solutions that are the most robust to changes in objective functions, identifying "elbows" in the Pareto frontier (i.e., where a small gain in one objective results in a large trade-off in another objective). The final step of choosing among the Pareto solutions requires problem-specific considerations that are subjective and depend on each decision. Ecological applications using the *a posteriori* approach include: Larkin et al. (2016) who used the weighted-sum method to identify Pareto optimal solutions for four objectives related to *ex situ* conservation of North American Angiosperms; Tsai et al. (2015) identified Pareto optimal solutions for reservoir operation with competing objective functions related to fish diversity and human satisfaction; Zhou et al. (2015) identified Pareto optimal solutions for water resource management with economic, environmental, and social objectives; Kennedy et al. (2008) found Pareto optimal solutions for the objectives of protecting habitat of an endangered species, protecting late successional forest reserves, and minimizing cost; and Groot et al. (2007) found Pareto optimal solutions for objectives related to financial returns from agriculture, landscape quality, nature conservation, and environmental quality.

4. Application of multi-objective optimization to select a population target for cackling geese

We consider the resource management problem of choosing a population target for cackling geese. The Pacific Flyway, in collaboration with the U.S. Fish and Wildlife Service, state and Canadian wildlife agencies, and representatives of Oregon and Washington farmers and Native Alaskan subsistence hunters, are in the process of revising the cackling goose management plan (Pacific Flyway Council, 1999). An important part of the plan is the population target. The population target is currently 250,000 birds. Cackling goose abundance is estimated annually using aerial survey data

(Williams, 2016). Annual hunting regulations are made based on comparing the population target to annual abundance estimates. If abundance estimates are above the population target, more liberal hunting regulations are selected. If abundance estimates are below the population target, more restrictive regulations are selected. That is, the hunting regulations are intended to keep abundance at or near the population target. In choosing a population target to implement in the next management plan, the Pacific Flyway must balance the competing interests of subsistence hunters in Alaska and private agricultural farmers in Oregon and Washington. Additional considerations the Pacific Flyway face include the satisfaction of recreational hunters, the ecological integrity of the community on the YK Delta, and other non-consumptive benefactors of the cackling goose population. We focus on the objectives related to subsistence hunters and farmers for this example. Based on these objectives, we first develop objective functions related to the management objectives. We then apply the weighted-sum method (with equal weights) to develop a composite objective function to maximize. We then examine post-optimization-specification of preferences by optimizing a large number of composite functions obtained from the weighted-sum method with varying stakeholder weights. Our aim was to demonstrate the applicability of MOO as a tool for selecting a population target rather than to provide a solution to the problem. Thus, we use variables in place of values specific to the problem.

4.1. Objective functions

An objective function $f_i(\theta)$ associated with the population target θ of cackling geese quantifies the value of each potential $\theta \in \Theta$ relative to the objective it represents. For pre-optimization specification of preferences, we consider three objective functions. Two objective functions, $f_s(\theta)$ and $f_a(\theta)$, represent the Pacific Flyway's competing objectives related to Native Alaskan subsistence hunters and agriculture in Oregon and Washington, respectively. The third objective function $f_b(\theta)$ is concerned with balancing the importance of the other two objective functions. For example, a management agency concerned with stakeholder satisfaction might consider it unacceptable to implement management actions that result in one stakeholder to be completely satisfied while the other stakeholder is completely dissatisfied. Thus the management agency might have an explicit objective of balancing the satisfaction among stakeholders (i.e., maintain harmony). All three objective functions depend on one variable θ, that represents the number of geese for the population target. We did not consider an objective function for recreational hunting, presuming that the objectives of recreational hunters would closely align with those of subsistence hunters.

The task of formulating objective functions is about the translation of an interest or aim to the formal language of mathematics (Hennig and Kutlukaya, 2007). There are an infinite number of possible objective functions that might be chosen. For pragmatic reasons, it is necessary to consider only a small number of possible objective functions (Hennig and Kutlukaya, 2007). Different decision makers may likely construct different objective functions for the same problem. In absence of existing objective functions, we developed the objective functions based on simple axioms and hypotheses as suggested in Williams and Hooten (2016). These objective functions do not necessarily represent the views of the associated stakeholders and are only used for demonstrating the MOO procedure. In applied settings, objective functions should be developed in close collaboration with resource managers and/or stakeholders, which is beyond the scope of this paper. Collaborating with these groups to develop objective functions helps ensure the objective functions accurately represent their belief. It also promotes acceptance of the decision analysis process. Without collaborative development of objective functions it is unlikely that managers/stakeholders will be satisfied with the solution, regardless of how closely the objective functions reflect their objectives. We refer readers to Keeney and Raiffa (1976), Gregory et al. (2012) for formal techniques for soliciting objective functions.

We developed the objective function associated with subsistence harvest based on three axioms. The first axiom was that if hunting regulations did not permit subsistence hunting, the value of $f_s(\theta)$ would be zero. The current management plan closes the hunting season when θ< 80,000 birds. We denote the value at which hunting is closed as χ for generality. The second axiom was that an increase in the management population θ was associated with larger values of $f_s(\theta)$. That is, more birds implied higher utility. The third axiom was diminishing marginal returns. When considering a very large population of geese (i.e., a population size θ in which a hunter providing reasonable effort could harvest as many geese as needed; e.g., 1 million birds) it is hard to distinguish between the utility of θ and $\theta + \tilde{\theta}$ (e.g., 1.01 millions birds). That is, the marginal utility of additional birds beyond θ is too small to be recognized by subsistence hunters. By the second axiom, $\theta + \tilde{\theta}$ is better than θ, but a large value of θ would already provide enough birds to maximize storage and consumption potential. The additional $\tilde{\theta}$ birds would provide little benefit, given the large population size of θ; $f_s(\theta)$ is bounded by some upper limit. The upper limit reflects maximum storage capacity and/or maximum consumptive capacity of subsistence hunters. The objective function should therefore be concave, increasing faster at low values of θ, then reaching an asymptote. We used Bernoulli's utility function for wealth to describe the value or utility of each population target θ (Eq. (4); Bernoulli, 1954). We selected an upper limit beyond the cackling goose carrying capacity, K, so the function was close to linear in the population range of $\chi, \ldots, K$ birds. A linear utility function would result in the largest population target of all concave functions, and thus is an optimal choice for subsistence hunters. Any other concave function would result in a sub-optimal choice of population target with respect to subsistence hunters. We scaled the resulting objective function between 0 and 1 for values of θ between 0 and K (Fig. 5A).

The impact of cackling geese on agriculture depends on both the number of birds in the population and the distribution of the wintering population relative to public and private land. The mechanisms that determine habitat use by cackling geese relative to public and private land and relative to abundance are unknown. We developed two hypotheses describing the extremes of the potential distribution of geese relative to public and private land with respect to abundance. The hypotheses were: (1) geese feed exclusively on private land and depredate crops proportionately to population size, and (2) geese first use public land until the number of geese approaches the public-land carrying capacity. The remaining geese use private land after the public-land carrying capacity is met and depredate crops proportionately to population size. We represented the public land carrying capacity with the symbol K_{pl}. We developed a negative linear function scaled between 0 and 1 to represent the first hypothesis ($f_{a,a}$; Fig. 5B). We developed a piece-wise linear function ($f_{a,b}$) that equaled 1 for $\theta = 0, a, \ldots, K_{pl}$ and decreased linearly for $\theta > K_{pl}$ with slope equal to the slope of $f_{a,1}$ for the second hypothesis. Assuming each hypothesis was equally likely, we averaged the two hypotheses to obtain the agricultural objective function ($f_a(\theta)$; Fig 5B).

Finally, we developed an objective function ($f_b(\theta)$) that represented balancing competing stakeholder interests. The objective function $f_b(\theta)$ had small values when $f_s(\theta)$ and $f_a(\theta)$ were far apart and large values (i.e., 1) when $f_s(\theta) = f_a(\theta)$. This objective function assumed equal attainment of each objectives ($f_a(\theta)$ and $f_s(\theta)$) is preferred to unequal attainment of each objective. We used the negative squared-error loss function, scaled between 0 and 1 for

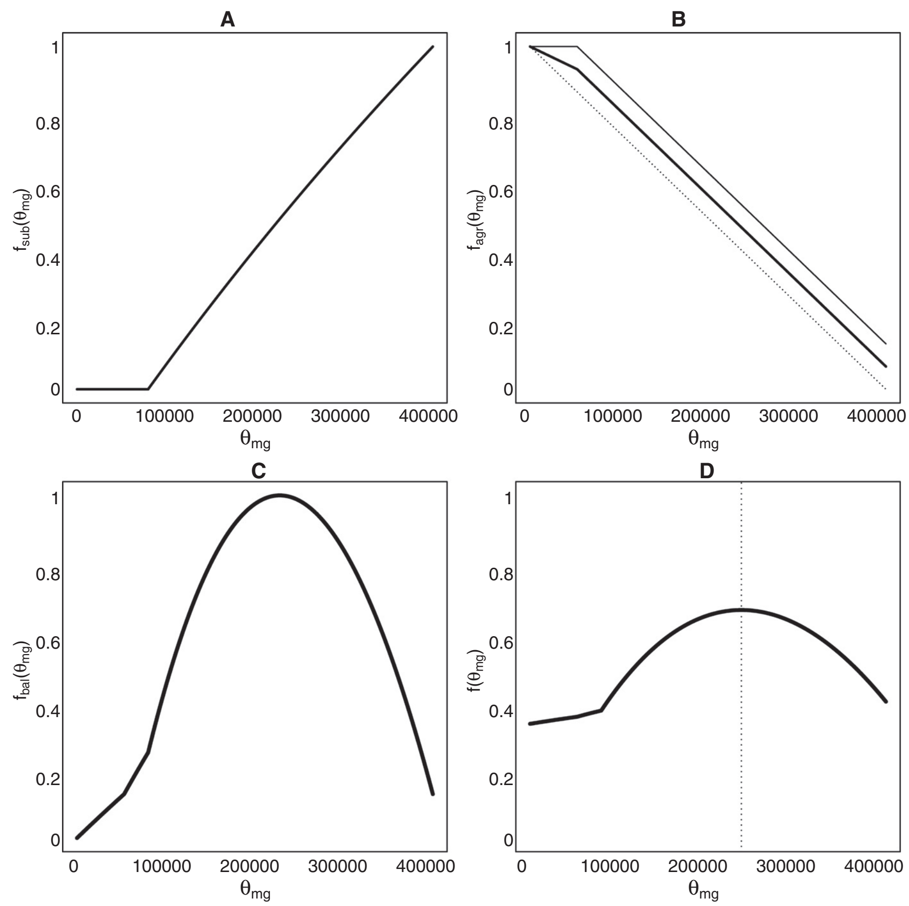

Fig. 5. Objective functions for selecting a cackling goose population target. (A) Objective function representing the objective of maintaining a large population size to improve subsistence hunting opportunities. (B) Objective function representing the objective of minimizing crop depredation in cackling goose wintering areas. The dashed line represents the hypothesis that cackling geese first use public land until the carrying capacity is reached (assumed to be 54,000 in this example), then decreasing proportionately to abundance. The dotted line assumes geese forage exclusively on private land. The solid line is the average of the dotted and dashed lines. (C) Negative squared error loss between $f_a(\theta)$ and $f_s(\theta)$, scaled between 0 and 1. (D) Weighted sum of $f_a(\theta)$, $f_s(\theta)$, and $f_b(\theta)$, with weights $\mathbf{w} \equiv 1/3$.

$f_b(\theta)$ (Fig. 5C). The mathematical form of each objective function was:

$$
\begin{aligned}
&f_s(\theta) = \begin{cases} 0, & \theta \le \chi \\ \log(b + c\theta), & \theta > \chi \end{cases}, \\
&f_a(\theta) = 1/2 f_{a,1}(\theta) + 1/2 f_{a,2}(\theta), \\
&f_{a,1}(\theta) = 1 - \frac{\theta}{\max(\theta)}, \\
&f_{a,2}(\theta) = \begin{cases} 1, & \theta \le K_{pl} \\ 1 - \dfrac{\theta - K_{pl}}{\max(\theta)}, & \theta > K_{pl} \end{cases}, \\
&f_b(\theta) = -(f_s(\theta) - f_a(\theta))^2,
\end{aligned}
\tag{4}
$$

for $\theta = 0, \ldots, K$. We scaled the functions $f_s(\theta)$, $f_a(\theta)$, and $f_b(\theta)$ between 0 and 1 using the equation $(f(\theta) - min(f(\theta)))/(max(f(\theta)) - min(f(\theta)))$.

4.2. Pre-optimization specification of preferences

Given the assignment of objective functions, any of the methods described in the previous section or in Table 2 could be used to incorporate preferences among objectives to identify an optimal solution. The non-decreasing and non-increasing nature of $f_s(\theta)$ and $f_a(\theta)$, respectively, make several of the MOO techniques trivial. The bounded objective function will result in an objective at the minimum or maximum bound conditional on which objective is ranked highest. The lexicographic method will result in an objective at either 0, max($f_b(\theta)$), or K, conditional on what objective is ranked highest. The Pareto set includes all $\theta = [0\text{–}K]$. We considered the weighted-sum method to identify an optimal population target.

Specifying weights for each objective is difficult when objectives represent stakeholder beliefs or values. Perhaps the most politically palatable set of weights for stakeholders with competing objectives are those in which $w_1 = w_2 = \cdots = w_k$ for k stakeholders. We assumed $w_1 = w_2 = w_3 = 1/3$ representing equal weight among stakeholders and equal weight to the penalty for differences in $f_s(\theta)$ and $f_a(\theta)$. The resulting objective function was:

$$
f(\theta) = \frac{1}{3} f_s(\theta) + \frac{1}{3} f_a(\theta) + \frac{1}{3} f_b(\theta). \tag{5}
$$

The weighted-sum method combined the multiple objective functions described in (4) to a single objective function that could be optimized using standard techniques (e.g., visually inspecting a plot). The resulting function $f(\theta)$ and the optimal solution θ^* are shown in Fig. 5D. The solution is sensitive to the choices of χ, K, K_{pl}, and the weights $\mathbf{w}$. The value χ is determined by hunting regulations and the easiest to identify (80,000 for cackling geese). The carrying capacity, K, could potentially be estimated using historical abundance data when available. The private land carrying capacity, K_{pl}, could be estimated using

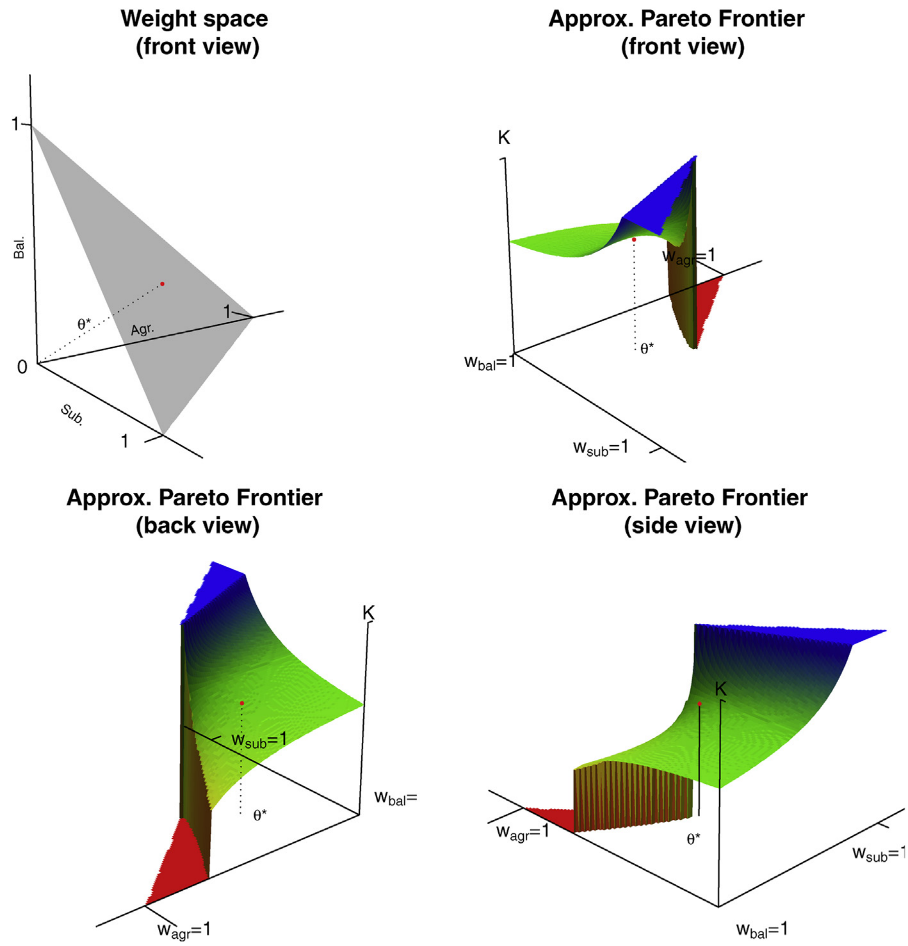

Fig. 6. (Weight space) Plane representing the possible combinations of w_i, $i = 1, 2, 3$ such that $\sum_{i=1}^{3} w_i = 1$ and $w_i \geq 0$ used in the equation $f(\theta) = \sum_{i=1}^{n} f_i(\theta) w_i$. The red dot indicates equal weights among the three objectives of the population target of balancing the competing objectives of subsistence harvest, agriculture, and balancing the first two objective functions. (Pop. target) Three angles of a surface plotted using the set of Pareto optimal solutions for a cackling goose population target obtained by optimizing the weighted objective function with varying values of weights w_i, $i = 1, 2, 3$ such that $\sum_{i=1}^{3} w_i = 1$. The *X*–*Y* plane of the three coloured figures is the weight space. The *Z*-axis is the value of the optimal population target for each combination in the weight space. The optimal solution obtained from equal weights = 1/3 is shown with the red point. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

bioenergetic requirements of cackling geese and the amount of food produced on available public land (e.g., McWilliams and Raveling, 2004). We developed a web-based application to explore the optimal population target in which the decision makers can identify values of χ, K, and K_{pl} relevant to the decision problem (https://perrywilliams.shinyapps.io/popobjective; Chang et al., 2016). We used R version 3.0.2 (R Core Team, 2013) to find optimal solutions of the combined objective functions.

4.3. Post-optimization specification of preferences

The population target θ^* shown in Fig. 5D is the Pareto optimal solution associated with the choice of weights $\mathbf{w} \equiv 1/3$. A decision maker might wish to view the sensitivity of the optimal population target relative to the choice of weights, or alternatively, view the implied weights of a given management objective. To examine these trade-offs, we calculated the Pareto optimal set and plotted each Pareto optimal solution with respect to the implied weights of the solution (Fig. 6). To calculate the Pareto optimal set we calculated optimal solutions of objective functions described by (5) with differing values of w_i, such that $w_i > 0$ and $\sum_{i=1}^{k} w_i = 1$ (i.e., values in the weight space; Fig. 6).

We plotted the optimal solution for each combination of weights in the weight space to examine how changes in preferences affect changes in the population target, and which population targets were relatively robust to changes in preferences (Fig. 6). Examination of Fig. 6 reveals several important points related to the information obtained using an *a posteriori* method that was not available using the *a priori* method. First, the objective weights of 1/3 applied to each objective function appears robust to small changes in $\mathbf{w}$ because it is on a relatively flat surface in Fig. 6. Thus, the choice of θ^* as a population target was relatively insensitive to the choice of weights. Second, as w_b approaches zero the small changes in the remaining weight assigned to w_a and w_s result in large differences in the resulting optimal population target. Third, as w_b approaches one, differences in weights have a small effect on the optimal population target. Thus, the most robust population target is one that has a large values of w_b. The second and third points illustrate the importance of an objective function that balances the competing objective functions; without it, the population target can reflect a large disparity between the implied weights given to each stakeholder; with it, the implied weights are relatively insensitive to the population target.

Given the trade-off solutions shown in Fig. 6, we considered four approaches for selecting a final population target. The first method was to choose the Pareto optimal solution that was the most common solution to $\theta^* = \max\{f(\theta)\}$ for all combinations of w_i, $i = 1, 2, 3$ such that $\sum_{i=1}^{3} w_i = 1$ (i.e., the weight space), excluding population

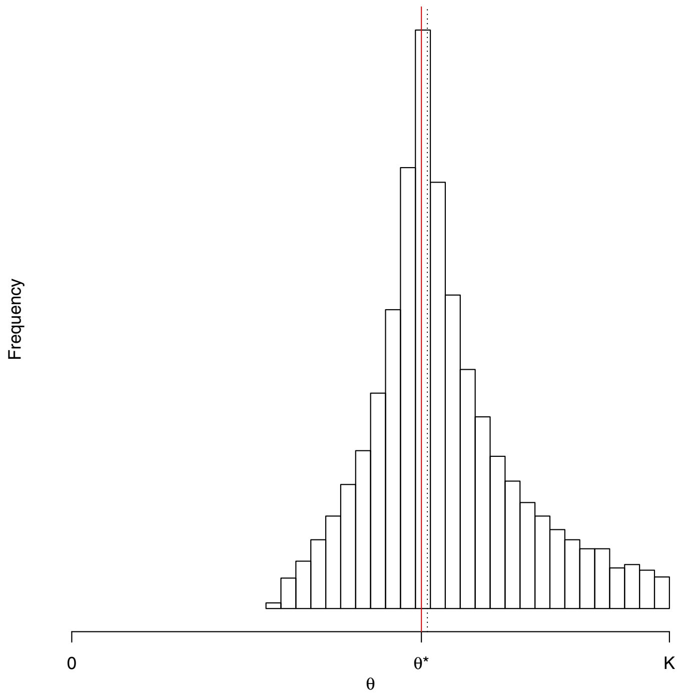

Fig. 7. Distribution of optimal population targets found using the weighted-sum method with varying weight combinations of w_i, $i = 1, 2, 3$ such that $\sum_{i=1}^{3} w_i = 1$ and $w_i \geq 0$ used in the equation $f(\theta) = \sum_{i=1}^{n} f_i(\theta) w_i$. The red vertical line represents the mode; the population target that was most often selected as optimal. Also shown (black vertical dotted line) is the Pareto optimal solution obtained using equal weights among objectives. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

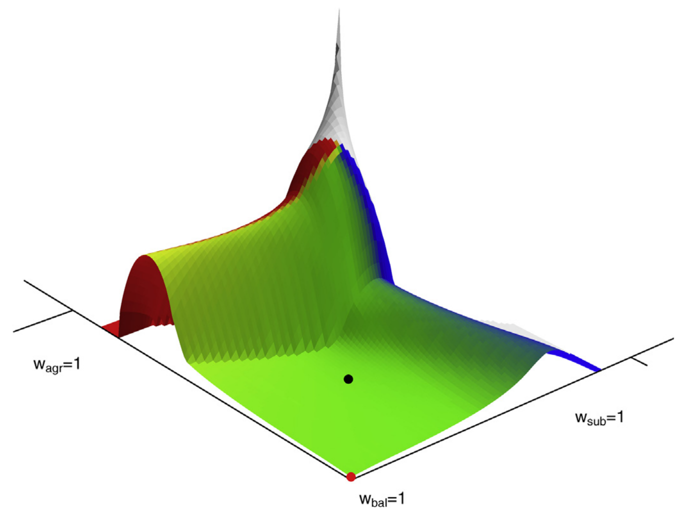

Fig. 8. Surface of the standard deviation (SD) of Pareto optimal population goals shown in Fig. 6 used as an indicator of robustness of the management decision to changes in objective weights. The SD was taken with respect to a 21×21 matrix of neighbouring values centred at each point with boundaries repeated ten times at each boundary. The optimal population target with the smallest standard deviation (red dot) occurs at $w_a = 0$, $w_s = 0$, and $w_b = 1$. Also shown (black dot) is the Pareto optimal solution obtained using equal weights. The colours used in Fig. 6 were maintained for reference. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

targets that were equal to 0 and K. The second approach was the population target that was most robust to change in stakeholder weights estimated using the standard deviation of 400 neighbors found using a 21×21 matrix centered at each point (for matrices that extended beyond the boundary, the boundary was used). The third approach was bounding $w_a < w_s$ and maximizing $f_a(\theta)$. Finally, the fourth approach was bounding $w_s < w_a$ and maximizing $f_s(\theta)$. The resulting optimal solutions are shown in Figs. 7–9. The first approach resulted in a solution similar to the solution obtained using the *a priori* approach with equal stakeholder weights (Fig. 7). The second approach also resulted in a solution similar to the solution obtained using the *a priori* approach with equal stakeholder weights, but implied different stakeholder weights. Specifically, $w_a = 0, w_s = 0$, and $w_b = 1$ (Fig. 8). The third approach resulted in a solution of $\theta^* = K$ (Fig. 8). This was due to Pareto optimal solutions occurring at K even when w_a was forced to less than w_s. This solution is not likely a viable option for cackling goose management. The fourth approach resulted in a solution at the same location in the weight space as the second option (i.e., $w_a = 0, w_s = 0,$ and $w_b = 1$; Fig. 8).

5. Discussion

We described three methods for optimizing a multi-objective problem using pre-optimization specification of preferences (the bounded objective function method, the lexicographic method, and the weighted-sum method). The combination of these methods or alternative methods described in Table 2 are applicable to a diverse array of problems. The optimal solution is sensitive to the choice of method. However, in a review of studies that used different optimization procedures in parallel for the same application, Huang et al. (2011) found the recommended course of action did not vary significantly with the method applied. We found evidence that different optimization procedures resulted in similar outcomes in the cackling goose problem of selecting a population target. The *a priori* strategy using the weighted-sum method with equal weights resulted in similar population targets to three of the four solutions obtained using the *a posteriori* strategy. However, one of the solutions from the *a posteriori* strategy resulted in a different solution; one that would not likely be acceptable from a management perspective, as the optimal solution was equal to the carrying capacity.

The weighted-sum method is a useful tool for both the *a priori* and *a posteriori* strategy. Variants of the weighted-sum method (e.g., simple multi-attribute weighting technique (SMART); Edwards, 1977) have been used in many ecological applications (e.g., Reynolds, 2001; Reynolds and Hessburg, 2005; Converse et al., 2013). The specific weighted-objective function we used for cackling geese is analogous to SMART (Edwards, 1977). However, the weighted-sum method is more general than SMART and is used in applications ranging from engineering (Marler and Arora, 2004) to statistical and mathematical tools including mixture models, Fourier transforms, and model selection. Using the weighted-sum method to identify the Pareto set is computationally straightforward.

The *a priori* strategy is more common in ecological applications than the *a posteriori* strategy. The *a priori* strategy is prescriptive; given preferences, a solution is prescribed. The *a posteriori* strategy does not require explicitly describing preferences; all preferences can be examined, and a choice implies preferences, but is based on the trade-offs observed. Deb (2001) argued that an *a posteriori* strategy is more methodical, more practical, and less subjective. He also concedes that if preferences can be reliably articulated, there is no reason to identify trade-off solutions. The *a priori* strategy is useful in decision problems consisting of stakeholders and/or decision makers that do not agree on a solution, but might agree on the inputs of a decision problem. For example, if the stakeholders associated with the cackling goose management problem agreed on values of χ, K, K_{pl}, $\mathbf{w}$ and their respective objective functions, and adhered to the decision analysis framework, an optimal population target could be prescribed. If they cannot agree on inputs into

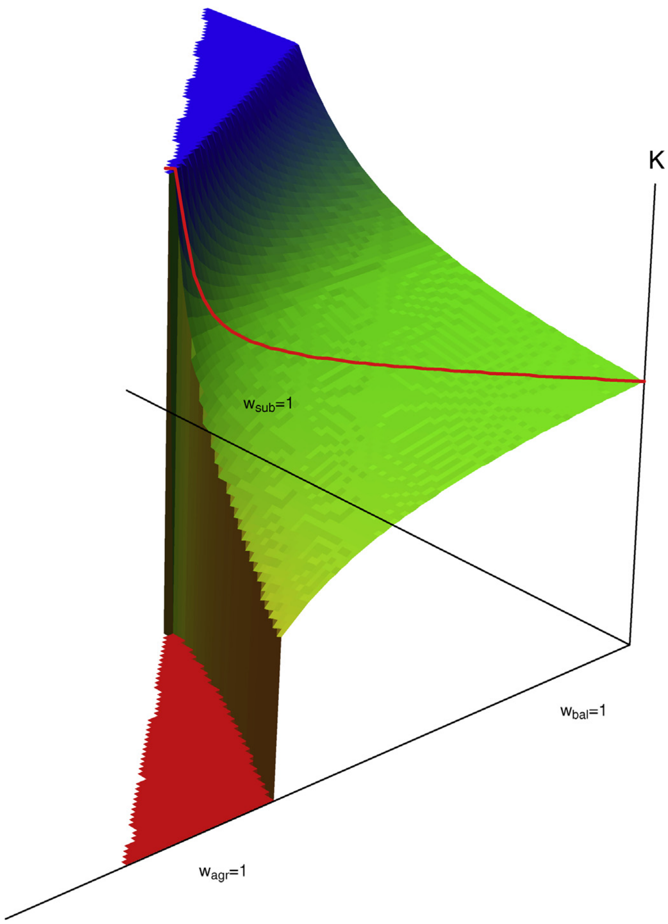

Fig. 9. Surface of Pareto optimal solutions for a cackling goose population target obtained by optimizing the weighted objective function with varying values of weights w_i, $i = 1, 2, 3$ such that $\sum_{i=1}^{3} w_i = 1$. The red line represents equal weights for the subsistence objective and the agriculture objective. Thus, for the decision rule that bounds $w_a < w_s$ and chooses the max with respect to $f_a(\theta)$ occurs at $\theta^* = K$. The decision rule that bounds $w_s < w_a$ and chooses the max with respect to $f_s(\theta)$ occurs at $w_a = 0$, $w_s = 0$, and $w_b = 1$; the same result that occurred using the standard deviation method (Fig. 8). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

the decision problem, decision making approaches other than MOO might be preferred (e.g., conflict resolution). Knowledge of consequences *via* an *a posteriori* strategy might bias a decision maker who desires a decision based on pre-optimization specification of preferences. These situations might result in a game-theoretic decision problem (Von Neumann and Morgenstern, 2007) instead of a MOO decision problem. That is, when competing stakeholders select objective functions or other inputs into the decision problem to optimize θ with respect to their interests, to the detriment of other stakeholders. Thus, an *a posteriori* strategy is likely more useful for a decision maker who has competing objectives but with non-competing interests. In these cases, knowledge of the trade-offs among Pareto optimal solutions might facilitate the decision process.

We focused on examples related to natural resource management. Several other applications of multi-objective optimization relevant to ecology include model selection (Williams, 2016), life history evolution (Kaitaniemi et al., 2011), and behavior ecology (Rothley et al., 1997; Schmitz et al., 1998). Model selection is concerned with balancing the competing objectives of model fit and model complexity (Burnham and Anderson, 2002; Williams, 2016). Numerous methods have been developed to address this problem, including the development of several information criteria (e.g., Akaike's information criterion, Schwartz' information criterion; Akaike, 1973; Schwarz et al., 1978). Information criteria are weighted averages of functions, heuristically related to model fit (usually assessed using model likelihoods) and model complexity. Williams (2016) examines model selection from a multi-objective optimization approach. Life history evolution concerns balancing vital rates to maximize fitness (e.g., clutch size vs. survival probability; Lack, 1947). Kaitaniemi et al. (2011) used multi-objective optimization as a mechanistic analysis of potential ecological an evolutionary causes and consequences of variation in life-history traits of a species of moth. Similarly, behavioral ecology concerns balancing competing interests such as finding food and avoiding predators (Mangel and Clark, 1986). Multi-objective optimization provides a basis for developing models to investigate hypotheses related to life history and/or behavioral ecology.

In addition to the two strategies we considered, there is an interactive approach to MOO (Shin and Ravindran, 1991). Interactive approaches consist of using an iterative solution algorithm to find solutions. After each solution is found, information is provided to the decision maker. The decision maker then uses the information and provides additional preference information. Given the additional preference information, another solution is found. Thus, the interactive approach allows the decision maker to adjust their preferences between each iteration and learn about the relationships between their preferences and solutions (Branke et al., 2008).

Although MOO methods can be used to address decision problems with a small number of alternative actions, there exists dedicated techniques for solving problems with a small number of alternatives (e.g., multi-criteria decision analysis). Triantaphyllou (2013) provides a comparison of these methods. For many problems, multi-criteria decision analysis might be more powerful than MOO methods.

Finally, although we limited the discussion of objective functions in this paper, appropriately modelling a decision maker's objectives is important for MOO to be useful, regardless of which strategy or optimization formula is used. Any optimal solution from a MOO problem is only as good as the objective functions describing the aims or interests of the decision maker. Analogous to statistical models of ecological processes, objective functions are models of decision maker preferences. As such, objective functions do not perfectly reflect the inherent objectives of the decision maker. However, parsimonious objective functions can be useful tools for facilitating complex decisions (Kendall, 2001; Williams and Hooten, 2016). Further, in the case of objective uncertainty (e.g., Fig. 5B), multiple objective functions/models could be considered with the goal of reducing uncertainty among objective functions through time. Methods developed for reducing ecological model uncertainty through time (i.e., adaptive resource management; Johnson et al., 1997; Nichols et al., 2007; Williams, 2012) could be employed for reducing objective function uncertainty through time.

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