A Spatio-Temporal Model for Ecological Colonizations

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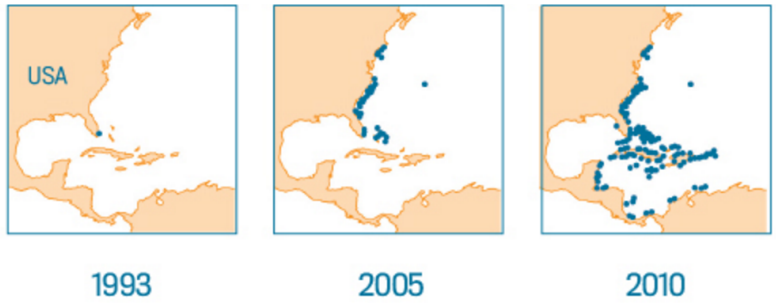
Background & Motivation



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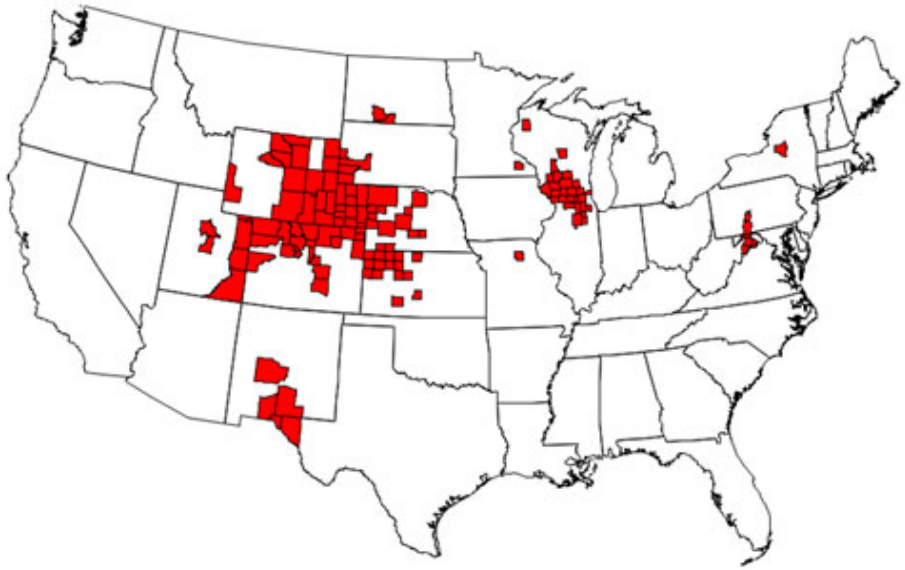

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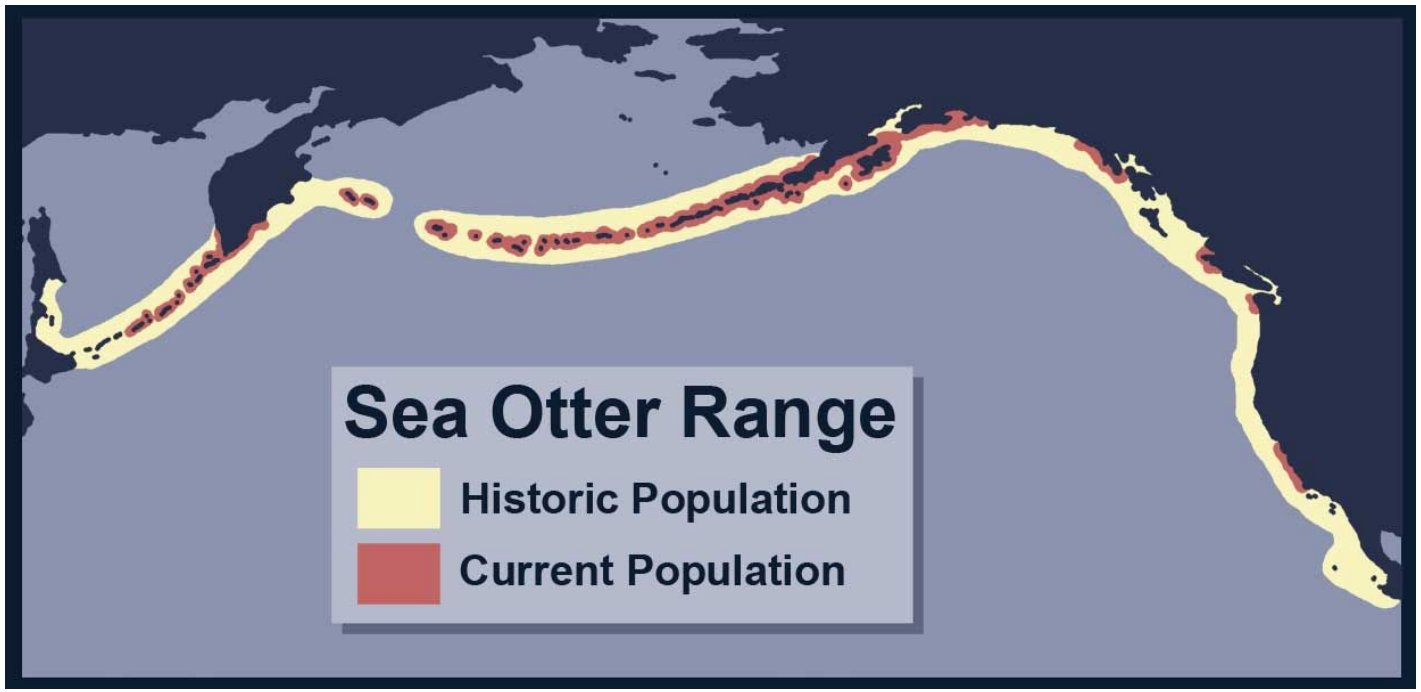
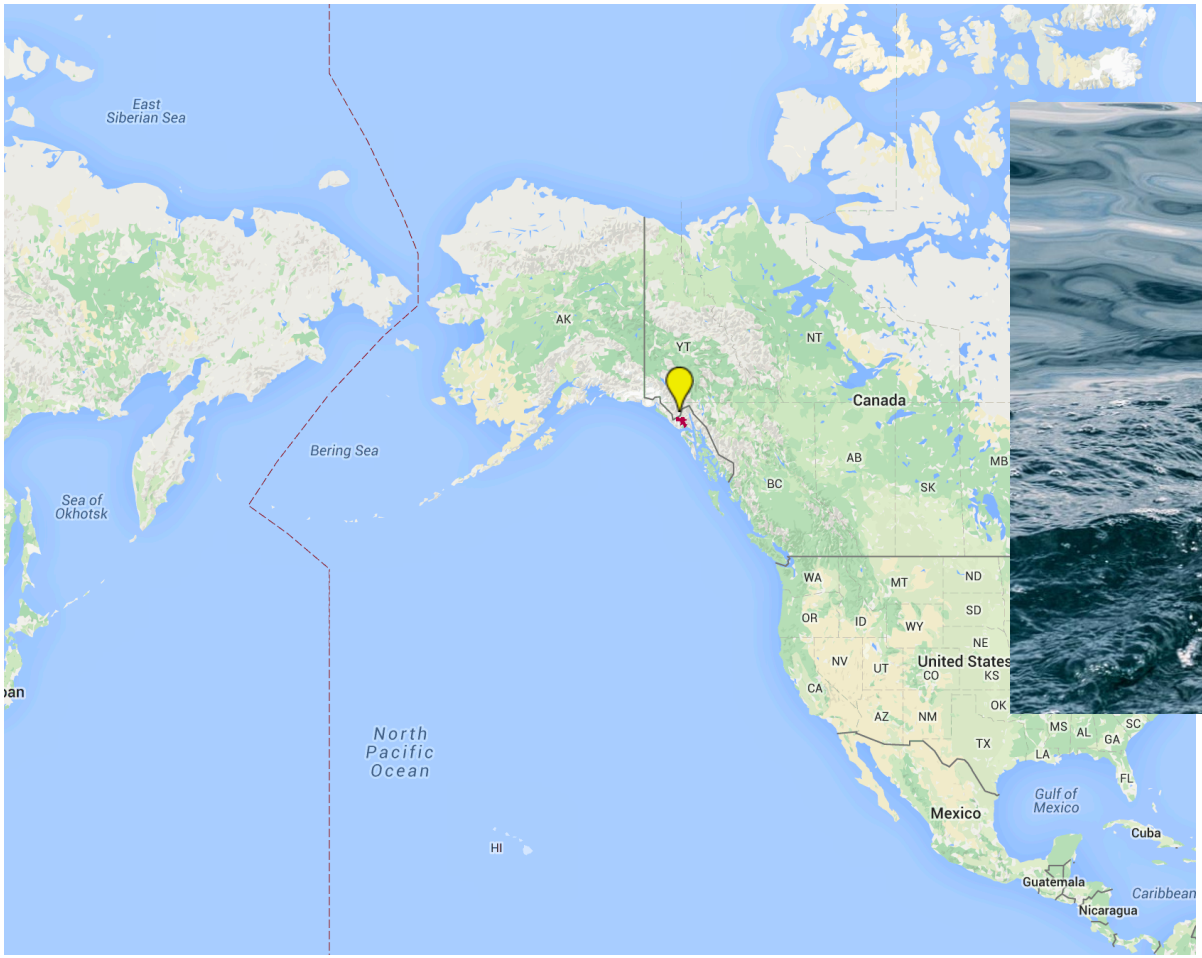


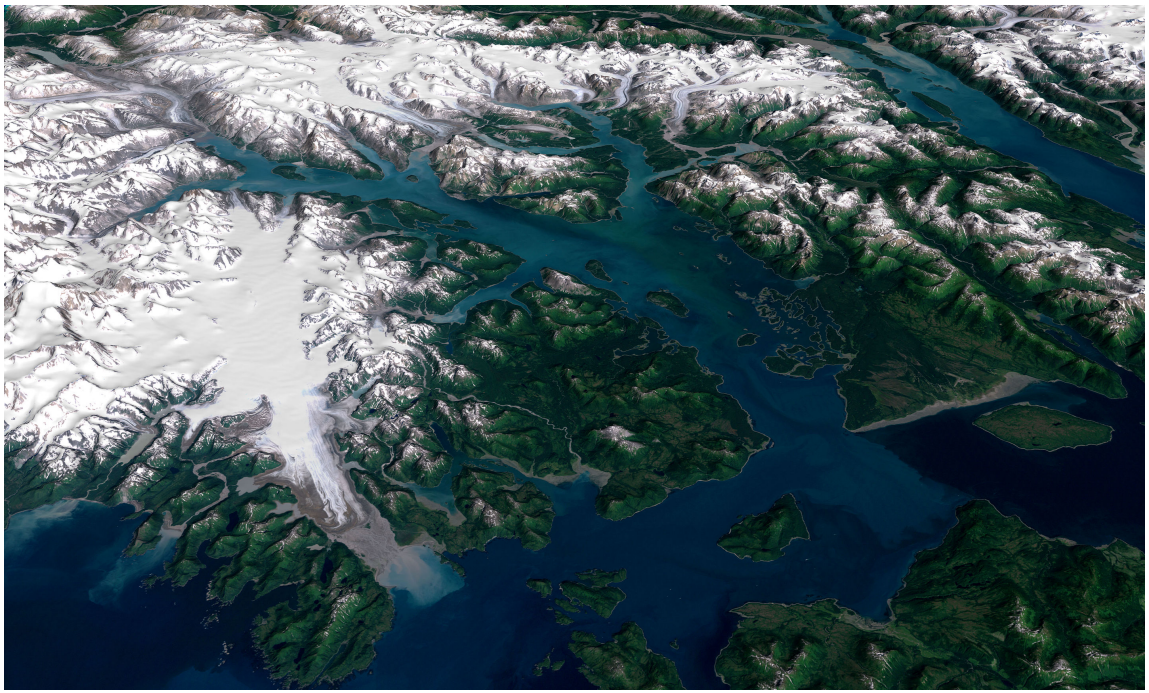
Background & Motivation

- Colonization/Invasion is a dynamic process.
  - Proliferation of statistical methods for modeling ecological dynamics (e.g., Wikle 2003, Wikle and Hooten 2006, Hooten et al. 2007, Cressie and Wikle 2011).
  - Incorporate models representing ecological theory and uncertainty.
- State variables of interest: distribution and abundance.

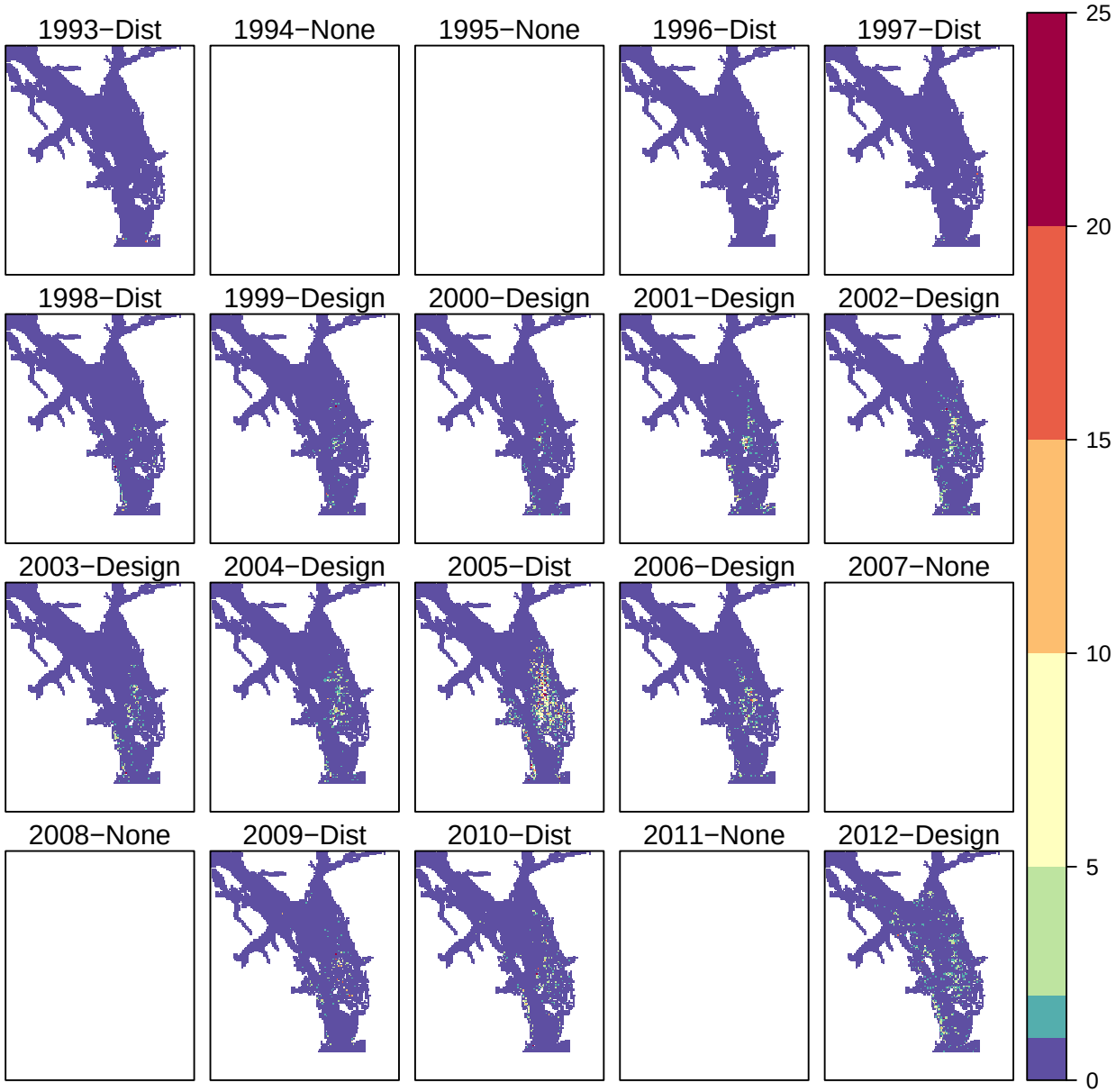
Motivating Ecological Application



Motivating Ecological Application



Data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Data Model

$$c_{i,t} \sim \begin{cases} \text{ZT-Binomial}(n_{i,t}, p_{i,t}), & y_{i,t} = 1 \\ 0, & y_{i,t} = 0 \end{cases}$$

$$y_{i,t} \sim \begin{cases} \text{Bernoulli}(\rho_{i,t}), & n_{i,t} \geq 1 \\ 0, & n_{i,t} = 0 \end{cases}$$

$$p_{i,t} = f(\mathbf{X}_p \boldsymbol{\beta}_p)$$

$$\rho_{i,t} = h(\mathbf{W}_\rho \boldsymbol{\alpha}_\rho)$$

$$\rho_{i,t} = 1 - (1 - p)^{n_{i,t}}$$

Process Model

$$n_{i,t} \sim \text{Poisson}(\lambda_{i,t})$$

$$\frac{\partial \lambda_i(\mathbf{s}_i, t)}{\partial t} = \left(\frac{\partial^2}{\partial s_{i,1}^2} + \frac{\partial^2}{\partial s_{i,2}^2} \right) \delta(\mathbf{s}_i) \lambda(\mathbf{s}_i, t) + \gamma(\mathbf{s}_i) \lambda(\mathbf{s}_i, t)$$

$$\lambda_i(\mathbf{s}_i, 0) = \frac{\theta e^{\frac{-|\mathbf{s}_i - d|^2}{\kappa^2}}}{\int_S e^{\frac{-|\mathbf{s}_i - d|^2}{\kappa^2}} ds}$$

$$\gamma(\mathbf{s}_i) = \alpha_0$$

$$\log(\delta(\mathbf{s}_i)) = \beta_0 + \beta_1(depth_i) + \beta_2(dist_i) + \beta_3(slope_i) + \beta_4(complexity_i)$$

Spatio-Temporal Occupancy Abundance Model

Data Model

$$c_{i,t} \sim \begin{cases} \text{ZT-Binomial}(n_{i,t}, p_{i,t}), & y_{i,t} = 1 \\ 0, & y_{i,t} = 0 \end{cases}$$

$$y_{i,t} \sim \begin{cases} \text{Bernoulli}(\rho_{i,t}), & n_{i,t} \geq 1 \\ 0, & n_{i,t} = 0 \end{cases}$$

$$\rho_{i,t} = 1 - (1 - p)^{n_{i,t}}$$

Process Model

$$n_{i,t} \sim \text{Poisson}(\lambda_{i,t})$$

$$\frac{\partial \lambda_i(\mathbf{s}_i, t)}{\partial t} = \left(\frac{\partial^2}{\partial s_{i,1}^2} + \frac{\partial^2}{\partial s_{i,2}^2} \right) \delta(\mathbf{s}_i) \lambda(\mathbf{s}_i, t) + \gamma(\mathbf{s}_i) \lambda(\mathbf{s}_i, t)$$

$$\lambda_i(\mathbf{s}_i, 0) = \frac{\theta e^{\frac{-|\mathbf{s}_i - d|^2}{\kappa^2}}}{\int_S e^{\frac{-|\mathbf{s}_i - d|^2}{\kappa^2}} ds}$$

$$\log(\delta(\mathbf{s}_i)) = \beta_0 + \beta_1(depth_i) + \beta_2(dist_i) + \beta_3(slope_i) + \beta_4(complexity_i)$$

$$\gamma(\mathbf{s}_i) = \alpha_0$$

Parameter Model

$$p \sim \text{Beta}(1, 1)$$

$$\boldsymbol{\beta} \sim \text{Normal}(\mathbf{0}, 10^2\mathbf{I})$$

$$\alpha \sim \text{Beta}(1, 1)$$

$$\kappa \sim \text{Normal}(5, 1)$$

$$\theta \sim \text{Normal}(500, 250^2)$$

Occupancy as a Derived Parameter

$$\phi_{i,t} = P(n_{i,t} > 0 | \lambda(\mathbf{s}_i, t))$$

$$= 1 - e^{-\lambda(\mathbf{s}_i, t)}$$

Results

Diffusion

- Negatively related to: shallow water, closeness to land, steep bottom slope, simple shorelines.

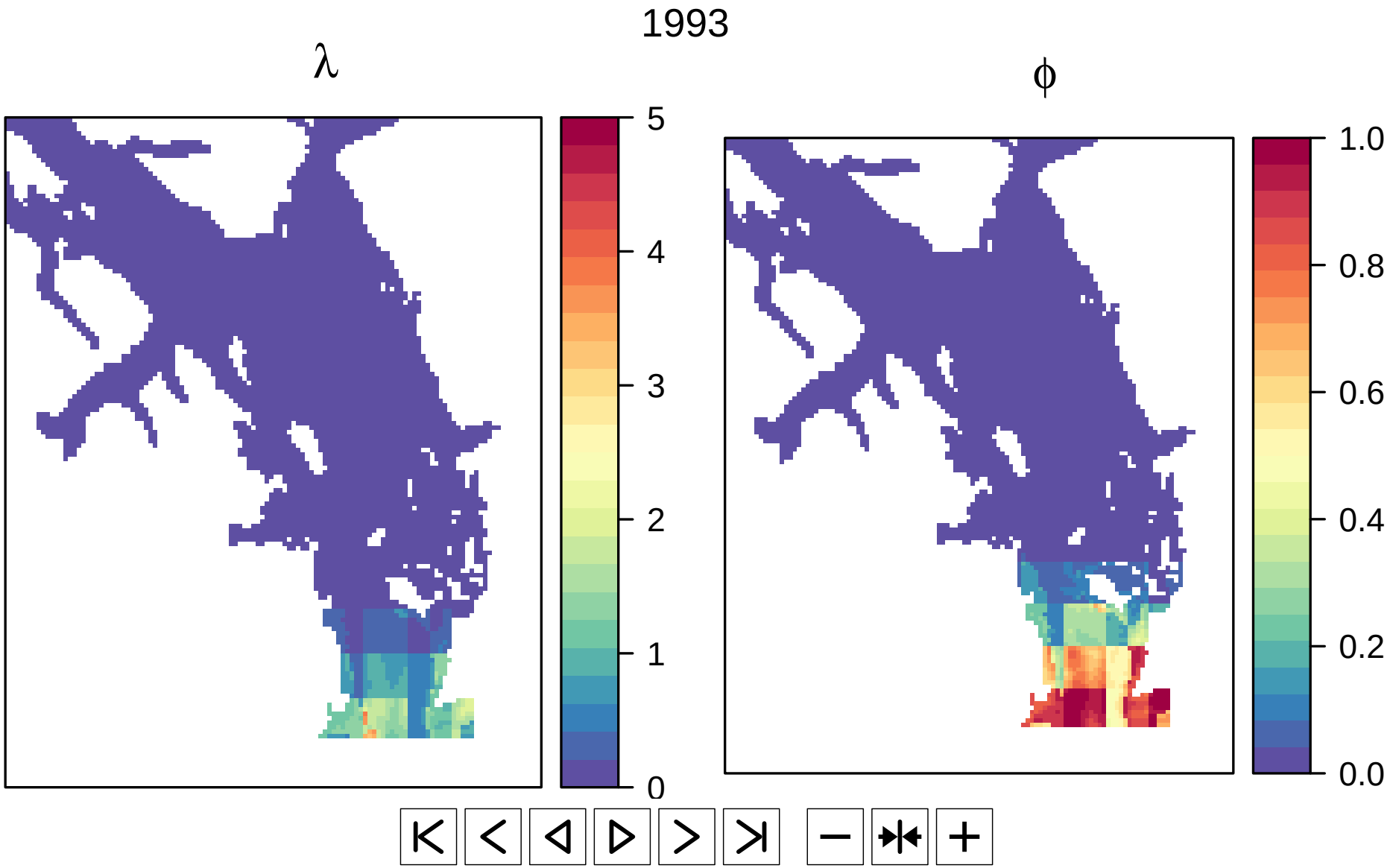
Growth

- 23.5% year^{-1}.
- Slightly exceeds other documented growth rates.

Occupancy

- Entire bay occupied as of 2016.

Abundance Intensity and Distribution



Summary

- Cohesive spatio-temporal modeling framework for estimating occupancy and abundance.
- Two stage detection process.
- Incorporates a theoretical model representing our understanding of the ecological system.
- Provides information for developing dynamic optimal sampling framework.

Williams et al. *In Revision*. An integrated data model to estimate spatio-temporal occupancy, abundance, and colonization dynamics. Ecology.